**Confidence Intervals**

Recall that the sample mean has a distribution with:

1. Mean . That is, when we take all possible samples of size *n* from a population, and calculate the mean of each of those samples, we will see that the mean of all those means, , will be equal to the population mean .
2. Standard deviation . That is,, the standard deviation of the sample mean distribution, is equal to the standard deviation of the population, , divided by the square root of the sample size *n*.

Also recall that the sample mean is the point estimator of the population mean *µ*, and the value is the point estimate of *µ*. However, it is often the case that statisticians want not just a *point* estimate of *µ*, but a *range* of values that *µ* can take on. That is, they want to say, with a certain degree of confidence, that *µ* will be within a certain interval (*l*, *u*), where *l* is the *l*ower bound and *u* is the *u*pper bound. Common levels of confidence are 90%, 95% and 99%. Specifically, we’re looking at a 100(1 – α)% confidence interval, where *α* is 0.1, 0.05, or 0.01, etc.

Note that in some situations, we are interested only in the *lower bound* or the *upper bound*, and not the whole interval. For example, if we want to say that *µ* will take on a value which lies in the interval (*l*, ∞) with 100(1 – α)% degree of confidence, then *l* is the *l*ower bound. Similarly, if we want to say that *µ* will take on a value which lies in the interval (-∞, *u*) with 100(1 – α)% degree of confidence, then *u* is the *u*pper bound.

Also, be sure to remember the proper interpretation of a confidence interval. For example, when α = 0.05 and we have a 95% confidence interval (*l*, *u*), it does not mean that P(*l* < *µ < u*) = 0.95. Instead, it means that if an infinite number of samples are drawn from the population and confidence intervals are created based on each of these samples, then 95% of these intervals will contain the true population mean, and 5% of these intervals will not. While we have no way of telling whether our particular sample will contain the true population means, we can say that if it does not, we’re pretty unlucky.

Finally, recall the relationship between confidence intervals and hypothesis tests. Assume we do the test at significance level α, and our null hypothesis is H0: . Then:

* In lieu of running a two-tailed test (Ha: ), we could calculate the two-sided 100(1 – α)% confidence interval.
  + If μ0 ≥ upper bound, or μ0 ≤ lower bound, we can reject H0 for Ha at significance level α.
  + If lower bound < μ0 < upper bound, we cannot reject H0 for Ha at significance level α.

***Case 1: Normal population, known***

*Notes: Relatively unrealistic situation, because is unknown in most cases.*

***Types of Confidence Intervals/Confidence Bounds***

1. ***Two-Sided Interval:***

*In Excel, obtain with:*

*=ABS(NORMSINV(α/2))*

*In R, obtain with:*

*abs(qnorm(α/2))*

***Case 2: Large n (≥40), unknown***

*Notes: N is large, so Central Limit Theorem (CLT) applies; X can have any distribution.*

***Types of Confidence Intervals/Confidence Bounds***

1. ***Two-Sided Interval:***

*In Excel, obtain with:*

*=ABS(NORMSINV(α/2))*

*In R, obtain with:*

*abs(qnorm(α/2))*

***Case 3: Normal pop., small n (<40), unknown***

*Notes: N is small, so CLT won’t apply, and we use a t-distribution.*

***Types of Confidence Intervals/Confidence Bounds***

1. ***Two-Sided Interval:***

*In Excel, obtain with:*

*=TINV(α, n-1)*

*In R, obtain with:*

*abs(qt(α/2, n-1))*

***\*\* Notice that in Excel we put in α, whereas in R we put in α/2***